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SIMPLE METHODS FOR CLUSTERING PROFILES AND LEARNING CURVES

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SIMPLE METHODS FOR CLUSTERING PROFILES AND LEARNING CURVES

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This report describes simple methods to test clusters in samples of N profiles or N learning curves. For sign patterns from k binary variables with probabilities ($p = 1/2$) of being plus or minus under homogeneity hypothesis ($\underline{H_0}$) $r = 2^k$ simultaneous binomial tests are suggested to detect clusters of patterns. Clusters are defined as patterns occuring more often that expected under $\underline{H_0}$. The main application is to $\underline{k}$ continuously distributed variables which have been dichotomized at their medians to give sign patterns. Clusters in this case correspond to clusters of profiles (= k-tuplets of measurements) of similar shape and level. When the k-point curves with $K = 2k$ as the number of observations per curve are dichotomized at their curve-medians then the curves may be represented by a sign pattern of an equal number of k plus signs and minus signs. Sign patterns more often observed that expected under the null hypothesis of no learning (horizontal curves) are called clusters of learning curves. The applications of the methods is illustrated by examples from (1) similarity judgements in a multidimensional scaling task and (2) from avoidance learning in gold fishes. The curve dichotomization test may be performed as an a priori or more usually, as an a posteriori test for detecting clusters of similarly shaped curves or profiles.

Keywords: Educational and psychological measurement, nonparametric method, cluster, profile, learning curve, panel judgement.

CONTENTS Sid

1. CLUSTERING PROFILES[1]

The ultimate goal of almost all research activities is to recognize and isolate patterns embedded in a homogeneous background. These patterns arise in experiments with k repeated measurements of N individuals (cases) or in scaling N stimuli by k judges. Typically the observations are arranged in two-dimensional matrices in order to simplify observations and judgements further. If the k univariate observations, called profiles below, are homogeneous, they may be represented by some average profile. If they are heterogeneous, the question arises how to detect clusters of homogeneous profiles. In this report a method is presented for clustering profiles as to their profile similarity. It may be considered as a special case of the one-sample configural frequency analysis (CFA) introduced by Krauth & Lienert (1975).

1.1 Homogeneity of profiles and cluster alternatives

A special case of categorial rating is half-and-half-rating. If n students, $N = 2n$, are rated for being above or below the median standard of a class in each of k tasks, the probability for either of the 2^k rating patterns is $p = 1/2^k$ under H_0 of a random half-and-half-rating. If there are clusters of like-rated students, they may be detected by a pattern dichotomization test (PDT). If the half-and-half-rating is made from N school reports according to the gradings in k tasks, clusters of alike-graded students may be defined. Half-and-half-rating may also be made by N students in the form of self-ratings. In this way, e.g., k social traits may constitute clusters of social behavior or social attitudes.

The case where one variable is rated by k raters half-and-half-rating is a special case of free-size-group rating of agreement between the k raters as to the N rated cases. If, e.g., the cases are adverbs (as in Chapter 1.2.1) the conditions for the example above are met.

To detect clusters of profiles or curves, the sample size in the PDT must be larger than in the curve dichotomization test (CDT) which is discussed in Chapter 2. For a given sample size, other things being equal, in general, the CDT is more efficient than the PDT.

If each of k variables, continuously distributed, is observed twice in N cases, a change may be in the positive (+) or negative (-) direction with equal probability ($p = 1/2^k$) under the null-hypothesis of no intervening factors between both observations in each variable. If any transfer or carry over effect is present, it may

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1. We are indebted to Dr. Sven Berg, Department of Statistics, University of Lund, Sweden, whose review and comments have done much to influence the final form of this report.

be general or differential (clusterwise). In the latter alternative a PDT of multivariate change is the most suitable testing. No change (0) is admitted in continuous variables and if it occurs in crude measurement, it must be solved by tossing a coin to give either plus or minus change.

1.1.1 An example from avoidance learning

For N = 32 learning curves of gold fishes, the PDT patterns were constructed according to the overall sample median. The 32 x 4 = 128 avoidance scores with $\bar{x} = 27.6$ as a cutting point are presented in the Appendix, Table 7. The observed distribution of pattern frequency is shown in Table 1.

Table 1. Observed pattern distribution and binomial tests

PDT-pattern	f	e	15	$(\frac{f}{N-f+1}) = F$	m; n
+ + + +	2	2			
+ + + -	0	2			
+ + - +	0	2			
+ + - -	0	2			
+ - + +	1	2			
+ - + -	0	2			
+ - - +	0	2			
+ - - -	0	2			
- + + +	12	2	15	8.571	42; 24
- + + -	0	2			
- + - +	0	2			
- + - -	0	2			
- - + +	5	2	15	2.678	56; 10
- - + -	0	2			
- - - +	6	2	15	3.333	54; 12
- - - -	6	2	15	3.333	54; 12
	N = 32	32			

In Table 1 the frequencies, f, of the $2^4 = 16$ possible patterns are given and the r = 16 binomial tests are carried out by means of F-tests since e = 2 is too small for z-tests. The F values of the f > e patterns are to be evaluated for m = 2 (32-f+1) and n = 2f degrees of freedom and for α^* . 05/16 = .003. Since F--tables for a protection level of .003 are not available the F's for df's not too small (as in Table 1) may be transformed into z-values by means of Paulson's formula (see Kendall & Stuart, 1965, p. 382).

$$z_F = \frac{F^{1/3}\left(1 - \frac{2}{9n}\right) - \left(1 - \frac{2}{9m}\right)}{\left[F^{4/3}\left(\frac{2}{9n}\right) + \frac{2}{9m}\right]^{1/2}} \tag{1}$$

For F = 8.57, $(F^{1/3} = (313.80)^{1/3} = 6.80$ and $F^{4/3} = (6.80)^2 = 46.17$ so that for pattern -+++ with F=8.57, m = 42 and n = 24)

$$z_F = \frac{6.80(1 - 2/9 \times 24) - (1 - 2/9 \times 42)}{\left[46.17\,(2/9 \times 24 + 2/9 \times 42)\right]^{1/2}} = +\,6.94 \qquad (2)$$

For F = 3.33 we get $F^{1/3} = (3.33)^{1/3} = 1.50$, and $F^{2/3} = (1.50)^2 = 2.23$ so that for m = 54 and n = 12

$$z_F = \frac{1.50\,(1 - 2/9 \times 13) - (1 - 2/9 \times 54)}{\left[\,2.23\,(2/9 \times 12) + 2/9 \times 54\right]^{1/2}} = +\,2.21 \qquad (3)$$

The z-value is associated with a p =.0136 which is larger than the required $\alpha^* = .003$. Thus the patterns with f = 6 are not significant, i.e. not constituting clusters. The only cluster-constituting pattern is -+++ with f = 12 which is significant at the .001 level of significance (see Fisher & Yates, 1957, Table V) and of course significant also at the level .003.

The interpretation of the significant pattern is that there is a cluster of cases being initially below the median score and later on above the median score of the total sample. Probably it is the type of learning curve with decelerating gain.

The mean learning curve of the cluster is achieved at by averaging the 12 curves of the cluster in Table 2.

The mean cluster curve is a learning curve raising from score 18 at the first trial to 34 and 36 in the next 3 trials. This may be some form of ceiling effect since the maximum score to be achieved were 40 in the cited learning experiment.

Table 2. Learning curves for the significant pattern in Table 1

16	33	37	38
17	38	34	36
12	38	37	38
21	40	35	38
22	38	33	36
26	38	33	38
14	35	35	28
23	30	36	29
19	34	32	38
27	31	36	34
8	28	33	38
6	28	39	40
17.6	34.2	35.0	35.9

1.2 Homogeneity of patterns and cluster alternatives

Consider a sample of N mutually independent observations x_{ij}, $i = 1(1)K$ and $j = 1(1)N$ where X is a continuously distributed variable underlying each profile. If the N cases or stimuli are coded as falling above (+) or below (-) an a priori defined cutting point of the variable X, the N patterns may be represented by any one of $r = 2^k$ pattern or signs. If the cutting point is chosen such that the probability of an observation $x_{ij} = +$ is 1/2, the probability of any pattern from k pattern points is 1/r under the null hypothesis (H_0) where H_0 implies that all patterns are equally likely with probability 2^{-k}.

If the researcher is interested in detecting alternatives for which some of the 2^k patterns are observed more frequently than expected under H_0 than according to CFA, he may perform r simultaneous binomial tests with parameters $p = 1/r$ and N using an $\alpha^* = a/r$ adjusted for r simultaneous tests. Exact tests may be performed by using suitable tables (e.g. Computation laboratory, 1955) or by using the statistic

$$F_i = \frac{f_i}{N - f_i + 1} \left(\frac{1 - p}{p}\right), \text{ where } p = 1/r \text{ and } i = 1 \ldots r \qquad (4)$$

which under H_0 is distributed as the F-distribution with $m = 2(N-f+1)$ df in the numerator and $n = 2f$ df in the denominator (see Pfanzagl, 1974, p. 116). In the case where N is large, and $p = 1/r$ is not too small, the normal approximation to the binomial may be used as an asymptotic test for detecting cluster of patterns, or sign patterns respectively, viz.

$$z = (f_i - e - .5) / \left[e(1 - e/N)\right]^{1/2}, \; i = 1 \ldots r. \qquad (5)$$

The frequencies f_i are the observed frequencies and $e = N/r$ is the frequency expected under H_0, correted for continuity. Only the patterns $f_i > e$ are of interest.

1.2.1 An example from multidimensional scaling

For developing a system for a computer-based content analysis of interview data (see Bierschenk & Bierschenk, 1976), k = 4 researchers were asked to judge N = 66 adverbs according to their similarity, and to form, according to their similarity judgement, groups of similar adverbs. The number of groups formed could range from 1 to N = 66. Under H_0 the assumption here is, that the grouping is random. That means (1) the number of groups of similar adverbs is a random number from 1 to 66 for each of four independent judges and (2) the allocation of an adverb to any of the k_i groups of judge i is a random allocation.

The agreement (+) and the disagreement (-) of the four judges in placing an adverb in one and the same group (judging it alike) is given in the Appendix, Table 6 for the adverbs in Swedish (originally) and in English translation.

As can be seen in the Appendix, Table 6 the judges 1 and 3 for example agree in placing the adverb 'how', while the judges 2 and 4 do not. The $r = 2^4$ resulting sign patterns possible are summarized in Table 3.

Table 3. Summary of sign patterns

Sign pattern	f	e = Np	z
++++	8	4,13	+ 1,63
+++-	12	4,13	+ 3,56
++--	2	4,13	- 1,27
+---	0	4,13	- 0,91
----	15	4,13	+ 5,01
---+	0	4,13	- 0,91
--++	4	4,13	- 0,30
-+++	4	4,13	- 0,30
+-+-	2	4,13	- 1,27
+--+	7	4,13	+ 1,14
+-++	3	4,13	- 0,79
-++-	7	4,13	+ 1,27
++-+	2	4,13	- 1,27
-+--	0	4,13	- 0,91
--+-	0	4,13	- 0,91
-+--	0	4,13	- 0,91
r = 16	66	66,00	$z_{(0,997)} = 2,73$

To detect cluster of patterns (H_1 to H_{16}), $r = 16$ simultaneous binomial tests were performed with parameters $p = 1/16$ and $N = 66$. The assumption under H_0 is that grouping for similarity is random thus giving expected frequencies of $e = 66/16 = 4.13$ and observed frequencies, f, as noted in column 2 of Table 3.

Since the expected frequencies are close to the minimum requirement, $e = 5$, normal approximation to the binomial tests were made using correction for continuity by subtracting .5 from $f-e > 0$. Two z-values in Table 3 are significantly above the alpha-adjusted z-value, $z_{.05/16} = z_{.003} = + 2.73$, thus two clusters of similarity judgement.

The one cluster of patterns with a sign combination +++- indicates that judges 1 to 3 have identical opinions on the classification of 12 specified adverbs while judge 4 has an opposite opinion on the same 12 adverbs (som helst, så värst, som möjligt, ..., något). A follow up of this cluster revealed that the researcher (No. 4) had not followed the instructions for this task but had grouped the adverbs according to their intensity.

The cluster with sign combination ---- contains 15 adverbs (mest, definitivt, alldeles, ..., mycket) for which none of the 4 judges could agree in their similarity. Therefore these 15 adverbs cannot be used for scaling purposes, while the 12 adverbs of the above cluster undoubtedly can be used.

In the case of k continuously measured variables each of k profile points is dichotomized as above (+) or below (-) the profile median. For simplicity it is assumed that the verbal and nonverbal IQs in WAIS of a sample of N = 32 students are under consideration. Furthermore, it is expected that the pattern ++, +-, -+ and -- are equally likely under H_0, where ++ is an IQ above the verbal and above the nonverbal median IQ. Applied to profiles (or learning curves) the pattern dichotomization test (PDT) is a means of clustering profiles as to their courses as well as to their levels.

The patterns ++ and -- in Table 1 are level patterns only while +- and -+ are course and level patterns: The first observation is above the population median (or some predefined cutting point on the scale) and the second observation is below. In this respect the PDT is a more informative clustering method than is the Curve dichotomization test (CDT), suggested in Chapter 2. However the number of possible patterns is larger in PDT than in CDT: 2^k versus $\binom{k}{k/1}$.

2. CLUSTERING LEARNING CURVES AS TO THEIR SHAPES

Tests for clustering sign patterns of k binary variables or profiles from batteries of k tests is proposed in Chapter 1. These tests cluster profiles as to similarity in levels and shapes by comparing the observed frequencies of the 2^k distinguishable patterns with a constant expected frequency.

For clustering learning curves the approach described in Chapter 1 has two disadvantages, namely

1. the number of patterns increases exponentially with the number k of curve points and
2. similarity in shape (important in learning curves) is confounded with similarity in level (unimportant in learning curves).

The latter disadvantage is shared with conventional methods of cluster analysis (see e, g, Rollett & Bartram, 1976).

In the following, a method will be presented where clustering is only due to shape of learning curves by dichotomizing them ipsatively (see Catell, 1946), and not normativley as in Chapter 1. Furthermore, the proposed method reduces the number of possible patterns from 2^k to k (k-1)/2, thus enabling us to cluster not only curves of longer range but also curves with a small number of k repeated measurements.

2.1 The curve dichotomization test

Consider a sample of learning curves from N individuals (cases) with K = 2k observations, x_{ij}, i = 1 (1) K and j = 1 (1) N, where X is a continuously distributed variable underlying each individual curve not necessarily identically distributed in the N cases. To dichotomize a curve ipsatively, rank its k observations as to their size and assign ranks 1 to k by a minus sign and k + 1 to K by a plus sign (or vice versa). Count then the number, f, of observed patterns for the number of possible patterns.

$$\binom{K}{k} = K!/k!\ (K - k!) = r \qquad (6)$$

The null hypothesis is that the N underlying curves are horizontal of arbitrary level. Thus the probability of any pattern being realized is given by

$$H_0 : p_1 = p_2 = \ldots = p_r = 1/r = p \qquad (7)$$

The hypothesis of horizontal learning curves is opposed to alternative hypothesis of the type $P_h > 1/3$ for at least one h (8)

These one-sided alternatives are called cluster hypothesis implying that at least one of the r sign patterns is realized more often than expected under H_0, where $e = N/k$ for all $h = 1 (1) r$ patterns.

To test for the above alternatives, r simultaneous binomial tests using the frequencies, f_h, could be performed with parameters p and N (see Lienert, 1973, Chap. 5). For protecting the preassigned alpha-level for simultaneous tests, an adjusted α-level is defined as $\alpha^* = \alpha/r$, as suggested by Krauth & Lienert (1973, Chap. 2). If N is large and p not too small such that $N_p > 5$, the binomial tests may be approximated by normal tests using continuity correction

$$z_h = (f_h - e - 1/2) / \left[e (1 - e/N)\right]^{1/2}, \quad h = 1 \ldots r, \tag{9}$$

where f is the observed frequency of a specified pattern, h_i. If z_h, the value of a sign pattern h is above its critical limit z_{α^*} the pattern h is called a cluster (of learning curves).

If N or p is small, exact binomial tests may be performed by means of computer programs of by F-test approximations. The curve dichotomization tests (CDT) presupposes learning curves with scores occur from discrete measurement of a continuous variable X, the respective cases could be removed from the sample, as has been done in the following example.

2.1.1 An example from avoidance learning

In a shock avoidance experiment 32 gold fishes were tested on k = 4 successive days in 40 trials each for shock avoidance. The avoidance scores of each fish and the respective CDT sign pattern are given in the Appendix, Table 8.

The $r = \binom{4}{2} = 4 \times 3 / 2 \times 1 = 6$ sign patterns possible are enumerated as f_h for each pattern, h_i, in Table 4, omitting the 3 median-tied curves marked by pattern (0000). The remaining N = 29 avoidance curves were tested for clusters by r = 6 binomial tests with parameters $p = 1/r = 1/6$ and N = 30, via z-tests.

Table 4. Summary of avoidance curves of gold fishes

h	pattern	f_h	$e = N/r$	$(f_h - e - \frac{1}{2}) / \left[e (1 - e/N)\right]^{1/2}$		$= z$
1	+ + - -	0	4.83	- 5.33/2.63	=	- 2.03
2	+ - + -	1	4.83	- 4.33/2.63		- 1.65
3	+ - - +	1	4.83	- 4.33/2.63		- 1.65
4	- + + -	2	4.83	- 3.33/2.63		- 0.89
5	- + - +	11	4.83	+ 5.67/2.63		+ 2.16
r = 6	- - + +	14	4.83	+ 8.67/2.63		+ 3.30*
		N = 29	28.98	$z_{0.05}/6$	=	+ 2.40

There is only one cluster --++ with x_6 = + 3.30 larger than the adjusted $z_{.008}$ = + 2.40 for the predisigned α = .05.

The possible r patterns of the CDT test represent ipsative types of shapes. Thus clusters must be represented by mean curves of ipsatively ranked learning curves constituting the respective cluster. With respect to the r = 6 sign patterns in Table 4, they may as clusters be interpreted as follows:
Clusters ++-- is a down-step curve where the first two points lie above the following two points. The cluster +-+- is an S-shaped curve and its complement -+-+ is an inversely U-shaped curve. The cluster +--+ and -++- are U-shaped and inversely U-shaped clusters. The only significant cluster in Table 4, --++ is an up-step cluster of a shape-type represented in Table 5 by the sum of ipsative ranks of the f_6 = 14 cases making up a cluster of avoidance curves.

Table 5. A significant cluster: Ipsative ranks of 14 gold fishes

1	2	3	4
1	2	3	4
1	2	3	4
1	2	3	4
1	2	3	4
1	2	3	4
1	2	3	4
1	2	3	4
1	2	3	4
1	2	3	4
2	1	3.5	3.5
2	1	3.5	3.5
1	2	3	4
1	2	3	4
16	26	46	52

Looking at the successive differences, 10, 20 and 8, the up-step cluster is a monotonic learning curve with main increase from second to third point of ipsative measurement.

2.2 Tests of predicted clusters

The curve dichotomization test (CDT) is an a posteriori method of detecting clusters of course-like curves. If there is any a priori information on clusters to search for, the CDT may be adapted to become an a priori method of clustering, as are Mosteller's tests of predicted order (see Sarris & Wilkening, 1977).

If, in the example above, the researcher had expected only two clusters, namely --++ and -+-+ where both are abstracted from test scores (e.g. are evi-

dent), two CDT tests had to be performed with $\alpha^* = \alpha/2 .05/2 = .025$ and $z_{\alpha^*} = + 1.96$. The respective $z_5 = + 2.16$ and $z_6 = + 3.30$ exceed the critical value of $z = + 1.96$, thus defining the predicted clusters.

While the cluster --++ is a trivial monotonic learning curve as seen above, the cluster -+-+ is a nontrivial curve. This curve is based on the theory that some fishes change their avoidance strategy from second to third trial block (day). If the number of predicted clusters, p, is smaller than the number of non-predicted clusters, $p < r$, the CDT test of predicted clusters is more efficient than the usual CDT, since alpha-adjustment must only be made for p simultaneous tests, $\alpha^* = \alpha/p$.

3. DISCUSSION

The pattern dichotomization test (PDT) is a special case of the more general configural frequency analysis (CFA) with the restriction that the k variables are to be dichotomized at their median. This procedure gives the $p = 1/2$ observations being above or below the median. By suitable designs, as in case of change or half-and-half-ratings in the PDT, the variables are a priori plus or minus with the probability of 1/2. All these PDT-tests are conditional upon $p = 1/2$ in the sample.

If the population parameters of variable (like IQ) are known, unconditional PDT-tests may be planned and performed by dichotomizing the observed variable values according to the population median score (IQ = 100). It is evident that clusters detected by unconditional PDT maybe pictured as clusters in the alternative population, from which the sample is a random sample.

As in CFA, dichotomization may be substituted by trichotomization. If k variables are trichotomized, then 3^k patterns occur. If trichotomization is made according to the sample tertiles (lower (-), medium (0) and upper (+) tertile), the probability under the null hypothesis of any pattern is $p = 1/3^k$. The respective tests for clusters would then be called pattern trichotomization tests (PTT). Rationale and execution is the same as in case of PDT with constant expectances ($e = N/3^k$) for a sample of N individuals observed on k continuous variables.

Principally, k variables may be polychotomized at their respective quantiles, but the number of patterns, g^k for g groups of N/g cases per group, is increasing exponentially. Thus the efficiency of detecting clusters decreases by the same degree.

The curve dichotomization test (CDT) seems to be sufficiently powerful if the departure from H_0 is of medium size, as in Table 4.

Following Cohen's (1969, p. 189) power table, such deviations have power indices of about .9 for N = 48 and $\alpha = .05$. The asymptotic power index, suggested by Bradley (1969, p. 125) is about .8 for whatever departures in large sample sizes and arbitrary protection level α.

The CDT is restricted to even-point learning curves. If $k^{+1} = 2m + 1$ is an odd point learning curve, the median point may be assigned a plus, thus giving $\binom{k+1}{m}$ sign patterns. To avoid working with odd-numbered curves, one may delete the least important curve point (initial, intermediate or terminal).

The CDT is suitable only for learning curves of relatively <u>short</u> range with k = 4 or 8. In curves of longer range successive pairs or triplets of observations may be pooled to give $K' = K/2$ or $K' = K/3$ superobservations. This procedure, however, implies that the individual level parameters change only slowly from

point to point of each learning curve.

If the alternative to H_0 is that all N learning curves belong to one and the same cluster not specified in advance, Friedman's (1940) test or, more efficiently, Kannemann's incidence trend test (Kannemann, 1975, 1976) is a better choice than the CDT tests in short-ranging curves.

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5. APPENDICES

5.1 Sign patterns from panel judgements

5.2 Sign patterns from avoidance experiments

5.1 Sign patterns from panel judgements

Table 6. Agreement between four judges: Similarity judgements and groupings of adverbs

Adverb		Judge			
Swedish	English	1	2	3	4
hur pass	how	+	-	+	-
minst	the least	-	-	+	+
lika	as ... as	+	-	+	-
bara (=endast)	just	-	+	+	-
allra	the very (of all)	+	-	+	-
mest	(the) most	-	-	-	-
som helst	ever so, very	+	+	+	-
så värst	(not) so	-	+	+	-
som möjligt	as possible	+	+	+	-
definitivt	definitely	-	-	-	-
alldeles	quite	-	-	-	-
rent	purely	-	-	-	-
helt	wholly	+	-	-	+
fullt så	(not) quite as	-	-	-	-
fullkomligt	entirely	+	-	-	+
fullständigt	completely	+	+	-	+
närmast	almost	-	-	+	+
nästan	nearly	-	-	+	+
näst intill		-	-	-	-
relativt	relatively	+	+	+	+
förhållandevis	proportionately	+	-	+	+
ganska	rather, pretty	+	+	+	-
tämligen	fairly	+	+	+	+
rätt		+	-	+	+
ungefär	about, roughly	-	+	+	-
så där	not very	+	-	-	+
i viss mån	to some extent	+	-	-	+
delvis	partly	+	-	-	+
mer eller mindre	more or less	-	+	+	-
mindre	less	-	-	-	-
möjligen	possibly	-	+	+	+
oändligt	infinitely	-	-	-	-
oerhört	enormously, awfully	-	+	+	+
tillräckligt	sufficiently	-	-	-	-
nog	enough	-	-	-	-
väl	rather too	-	-	-	-
så	so, such	-	-	+	+
så pass	that	-	-	-	-
för	too	-	-	-	-
alltför	(far) too	-	-	-	-

Table 6. Cont.

direkt	quite	+	+	+	-
precis	exactly	+	+	+	-
verkligt	(really) properly	+	+	-	-
riktigt	really, quite	+	+	+	+
verkligen	(really) indeed	+	+	-	+
adekvat	adequately	+	-	+	+
kolossalt	enormously	-	+	+	+
enormt	tremendously	-	+	+	+
väldigt	awfully	-	+	+	-
särskilt	particularly (not so much)	-	+	+	-
betydligt	considerably	+	-	-	+
synnerligen	extremely, most	+	-	-	+
mycket	very, much	-	-	-	-
mångfaldigt	many times	-	+	+	-
större	more	+	+	+	-
mer		+	+	+	-
jäkla	damn	+	+	-	-
djävligt	bloody	+	+	+	+
fruktansvärt	terribly	+	+	+	+
hemskt	awfully	+	+	+	+
vansinnigt	madly	+	+	+	+
icke	non	+	+	+	+
inte	not	+	+	+	-
ej	not	+	+	+	-
lite	some, somewhat	+	+	+	-
något	a bit	+	+	+	-

+ agreement in similarity judgement
- non-agreement in similarity judgement

5.2 Sign patterns from avoidance experiments

Table 7. Frequency data and learning curves of gold fishes*

Learning curve				PDT-pattern	Learning curve				PDT-pattern
1	0	0	0	- - - -	1	3	1	6	- - - -
0	2	12	28	- - - +	26	38	33	38	- + + +
6	17	10	28	- - - +	28	35	37	35	+ + + +
16	33	37	38	- + + +	14	35	35	28	- + + +
11	17	14	23	- - - -	23	30	36	29	- + + +
6	27	32	36	- - + +	18	26	23	31	- - - +
17	38	34	36	- + + +	19	34	32	38	- + + +
12	38	37	38	- + + +	6	3	0	5	- - - -
14	21	29	33	- - + +	27	31	36	34	- + + +
1	27	24	28	- - - +	29	36	40	38	+ + + +
21	40	35	38	- + + +	7	17	31	36	- - + +
22	38	33	36	- + + +	29	26	32	32	+ - + +
0	23	24	37	- - - +	12	10	27	27	- - - -
13	16	16	29	- - - +	8	28	33	38	- + + +
0	19	34	37	- - + +	6	38	39	40	- + + +
11	26	40	39	- - + +	9	3	8	6	- - - -

* Adjusted from Huber, Tables 19-22, Appendix

Table 8. Avoidance scores and sign patterns of gold fishes*

x_1	x_2	x_3	x_4	CDT-pattern	Rank-order
1	0	0	0	0 0 0 0	
0	2	12	28	- - + +	1 2 3 4
6	17	10	28	- + - +	1 3 2 4
16	33	37	38	- - + +	1 2 3 4
11	17	14	23	- + - +	1 3 2 4
6	27	32	36	- - + +	1 2 3 4
17	38	34	36	- + - +	1 4 2 3
12	38	37	38	- + - +	
14	21	29	33	- - + +	1 2 3 4
1	27	24	28	- + - +	1 3 2 4
21	40	35	38	- + - +	1 4 2 3
22	38	33	36	- + - +	1 4 2 3
0	23	24	37	- - + +	1 2 3 4
13	16	16	29	0 0 0 0	
0	19	34	37	- - + +	1 2 3 4
11	26	40	39	- - + +	1 2 3 4
1	3	1	6	- + - +	
26	38	33	38	- + - +	
28	35	37	35	0 0 0 0	
14	35	35	28	- + + -	
23	30	36	29	- + + -	1 3 4 2
18	26	23	31	- + - +	1 3 2 4
19	34	32	38	- + - +	1 3 2 4
6	3	0	5	+ - - +	4 2 1 3
27	31	36	34	- - + +	1 2 3 4
29	36	40	38	- - + +	1 2 3 4
7	17	31	36	- - + +	1 2 3 4
29	26	32	32	- - + +	
12	10	27	27	- - + +	
8	28	33	38	- - + +	1 2 3 4
6	28	39	40	- - + +	1 2 3 4
9	3	8	6	+ - + -	4 1 3 2

* Adjusted from Huber, 1972, Tables 19 - 22, Appendix

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Abstract card

Bierschenk, B. & Lienert, G. A. Simple methods for clustering profiles and learning curves. Didakometry (Malmö, Sweden: School of Education), No. 56, 1977.

This report contains a description of simple methods to test clusters in samples of N profiles or learning curves. Clusters are defined as patterns occuring more often than expected under the null hypothesis. The application of the methods is illustrated by examples from (1) similarity judgements in a multidimensional scaling task and (2) from avoidance learning in gold fishes.

Keywords: Educational and psychological measurement, nonparametric method, cluster, profile, learning curve, panel judgement.

Reference card

Bierschenk, B. & Lienert, G. A. Simple methods for clustering profiles and learning curves. Didakometry (Malmö, Sweden: School of Education), No. 56, 1977.

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